ANL-HEP-CP-98-65

Domain wall fermions at finite temperature.

J.-F. Lagaë[a*] and D.K. Sinclair[a*]

[a]HEP Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne IL 60439, USA

We investigate the properties of domain wall fermions on a set of quenched configurations at non-zero temperature. In particular, we compute the low lying eigenvalues of the DWF operator and study their relation with topology, level crossings and chiral symmetry breaking. We also measure the screening correlators and discuss the dependence on the extent of the extra dimension and the quark mass.

1. Introduction

An interesting aspect of the Domain Wall Fermion (DWF) formulation is that it provides a consistent framework for studying the chiral limit $m_q \to 0$. In particular, it is expected to satisfy the Atiyah-Singer index theorem. This is a distinct advantage over the Wilson and staggered formulations which are well known to suffer from shifts of the low-lying eigenvalues in the real and imaginary directions respectively [1]. However, the DWF formulation accomplishes this feat at the expense of introducing a 5^{th} dimension and, strictly speaking, all the nice properties are only recovered in the limit $N_5 \to \infty$ (overlap formalism). A careful (non-perturbative) study of the convergence is therefore needed before practical simulations can be carried out. In this study, we initiate such a program in the context of QCD at non-zero temperature (thereby continuing our effort to understand the link between topology and the chiral phase transition [2]). We have used for our computations a set of quenched configurations on a $16^3 \times 8$ lattice. The β values [and number of configurations] studied are: 6.2[170], 6.1[170], 6.0[100], 5.9[100] and 5.8[100]. These configurations were used earlier to compute the screening correlators and the low-lying eigenvalue spectrum with a staggered fermionic action [2]. Here, we first study the level crossings of the hermitian Wilson operator (in a manner similar to [5]), then move on to compute the spectrum of the DWF operator (also properly hermitized). The DWF calculations are carried out (so far) at $N_5 = 4$, 6, 8 and 10. The link between topology, the number of level crossings and the number of DWF eigenvalues which vanish (exponentially) wiht N_5 can then be studied. As was noted earlier for staggered fermions [2], the knowledge of the low-lying eigenvectors is also quite valuable since in the high temperature phase, their contributions dominate the disconnected mesonic correlators. We confirm this again for DWF and use it to improve the convergence of our conjugate gradient inverter and to lower the error bars on the disconnected correlators.

*Supported by DOE contract W-31-109-ENG-38.

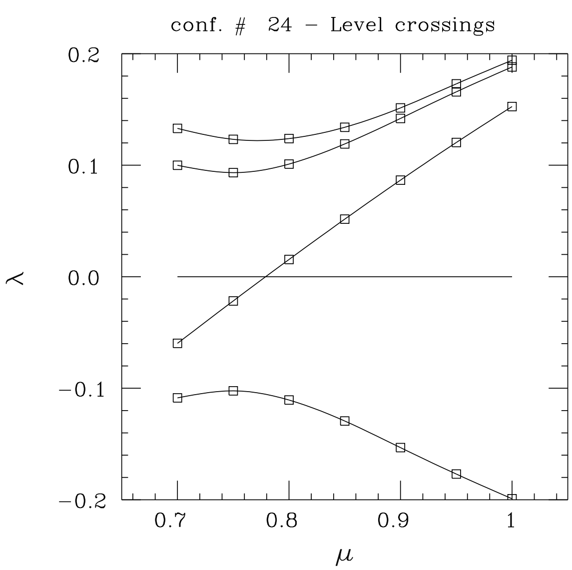

Figure 1. Level crossing of the hermitian Wilson operator on a configuration with topological charge 1.

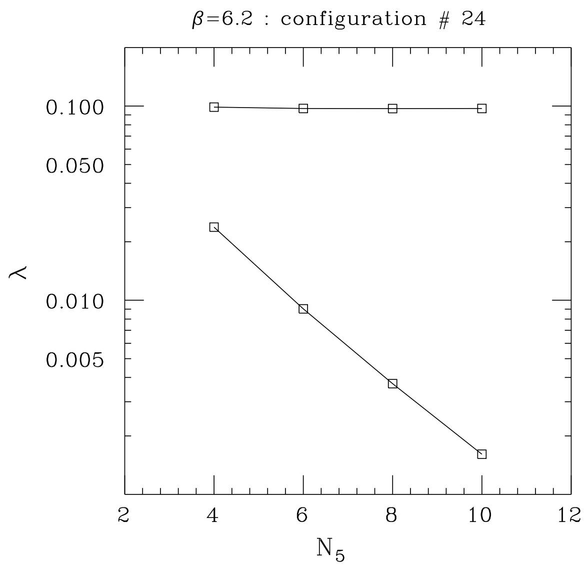

Figure 2. Eigenvalues of the hermitian DWF operator vs. N_5 on a configuration with $Q_{top} = 1$.

2. Results

In this preliminary report, we focus our attention on the results obtained at $\beta = 6.2$ (the highest temperature we have studied). The complete results, including the trends observed as one lowers the temperature towards the chiral phase transition will be reported elsewhere [3]. Also because of lack of space we illustrate the various computations on a single configuration. We have selected a configuration which has topological charge 1. We then obtain one level crossing in the spectrum of the hermitian Wilson operator (Fig. 1). It occurs at $\mu \approx 0.78$ with a slope of 0.74. On our sample of configurations, most crossings occured between $\mu = 0.75$ and $\mu = 0.80$. One notable exception was the single configuration of topological charge 2 for which the second crossing occured at $\mu \simeq 0.95$. There is a relationship between the presence of these crossings and the existence of zero modes of the hermitian operator $H \equiv \gamma_5 RD$ (where D is the DWF operator and R is the reflection operator in the 5^{th} dimension). For example, a crossing at μ with slope ± 1, would imply an exact zero-mode of H at a domain wall mass $M = 1 + \mu$. Around this value and for slopes of magnitude less than 1, there will be near zero-modes. In this study we have fixed $M = 1.7$ (similarly to what has been done in zero temperature applications [4]). On the same configuration as before, We then find that the lowest eigenvalue of H decreases exponentially with N_5, whereas the second lowest is essentially independent of N_5 (Fig. 2). The lowest eigenvalue is of order 0.0016 at $N_5 = 10$ and would be 2 order of magnitudes smaller if N_5 was extended to 20.

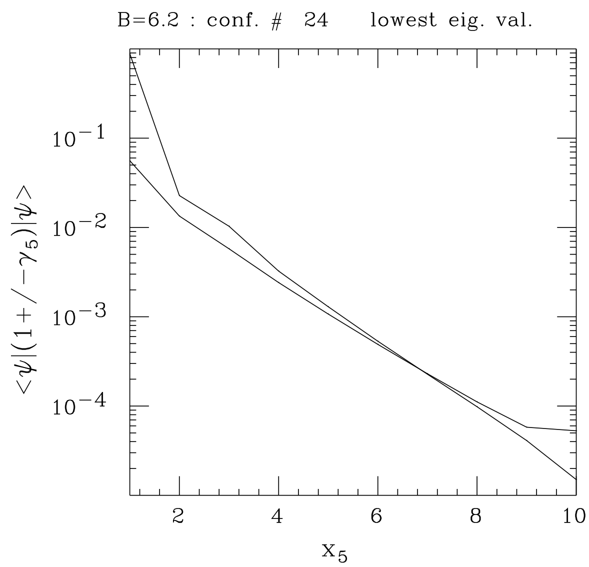

Figure 3. 4-dimensional slices of the “Left” and “Right” components of the lowest eigenmode

Our variational method for computing the eigenvalues also gives us the eigenvectors. Fig. 3 represents the positive and negative chirality components measured on the lowest mode as a function of x_5. The negative chirality component is peaked on the left wall (as should be for a configuration with $Q_{top} = -1$) and decreases exponentially in the interior. The right chirality component also reaches its maximum on the left wall but with a smaller magnitude. For modes with no net chirality, one would obtain a symmetrical graph with each component peaked on their respective wall. (This is what is observed for the “first excited state” on our sample configuration

and also for the lowest mode on configurations with $Q_{top} = 0$). On a few "abnormal" configurations with very late crossings [5], we found eigenmodes with a net chirality but an eigenvalue which remains large at $N_5 = 10$. A detailed discussion of this (small) artifact is left for [3].

Finally, as an introduction to our computation of disconnected mesonic correlators [3], we present our results for the measurement of m $Tr\gamma_5 S$ as a function of m at $N_5 = 10$. The continuum answer should be equal to the topological charge of the configuration (-1 in our case). At low values of the quark mass, fig.4 shows that the measurements dip towards 0. This is as expected for a situation where the lowest eigenvalue is not quite 0 yet at $N_5 = 10$. It is also important to note that a precise measurement of this operator was made possible by "separating out" the contribution from the lowest eigenmodes in the conjugate gradient inversion. Good results could then be obtained by using only 10 noise vectors on each wall. Without this "trick", even increasing the number of noise vectors by a factor of 10, still leaves us with rather large error bars (see Fig. 5).

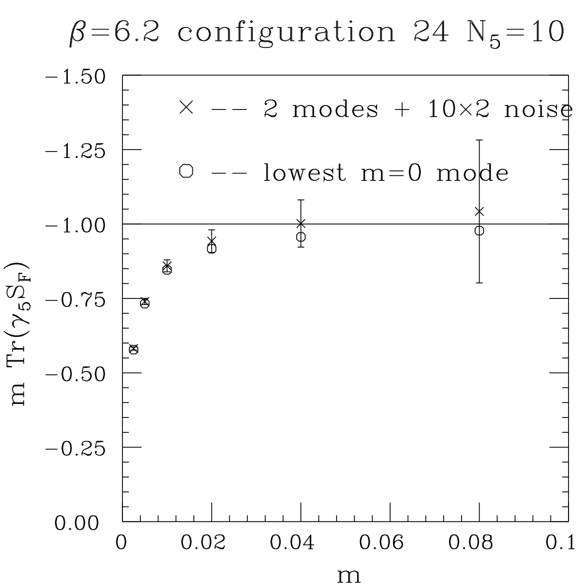

Figure 4. Chirality versus quark mass (using our knowledge of the low-lying modes)

3. Conclusions

We have confirmed (non-perturbatively) that DWF provide a practical and systematically improvable way of studying the topological properties of QCD (at least in the high temperature phase). The "rate of convergence" at $\beta = 6.2$ can be read off from Fig.2 (and similar results at lower β will be presented elsewhere). In addition, we have shown how the knowledge of the low-lying eigenmodes can be used to greatly improve the quality of fermionic measurements on configurations with non-trivial topology. We also expect that enhancements of this technique will play an important role in the design of dynamical fermion algorithms at very low quark masses.

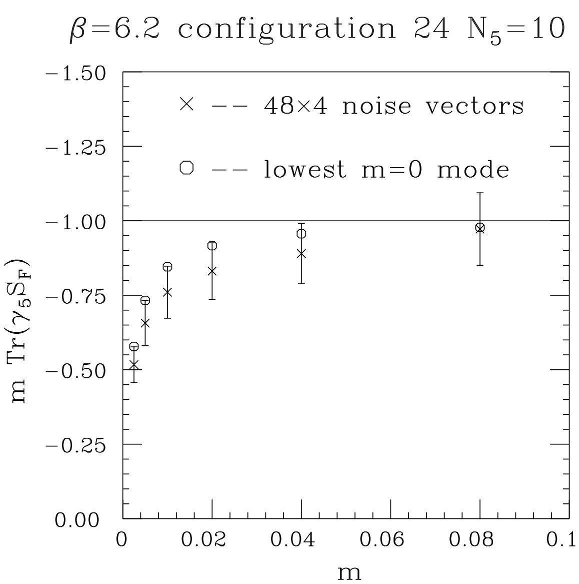

Figure 5. Chirality versus quark mass (noisy estimator only)

REFERENCES

1. J.C.Vink, Phys. Lett. **210**B (1988) 211
2. J.B.Kogut et al., Phys.Rev.**D58**:054504,1998.
3. J.-F.Lagaë and D.K.Sinclair, in preparation.
4. T.Blum, this volume.
5. R.G.Edwards et al., hep-lat/9802016.